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AN EXPLANATION OF WASHABILITY CURVES FOR THE INTERPRETATION OF FLOAT-AND-SINK DATA ON COAL

JAN 31

BY

G. D. Coe

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UNITED STATES DEPARTMENT OF THE INTERIOR - BUREAU OF MINES

AN EXPLANATION OF WASHABILITY CURVES FOR THE INTERPRETATION OF FLOAT-AND-SINK DATA ON COAL[1]

By G. D. Coe[2]

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INTRODUCTION

Float-and-sink tests indicate what results are obtainable by gravity separation and are therefore an invaluable adjunct to most coal-cleaning processes. However, it is difficult to make a direct interpretation of float-and-sink data on coal, and several systems for interpreting them have been proposed. Of these, that developed and used extensively by Bird[3], Yancey[4], and Gandrud[5] is considered by the author to be the most satisfactory. It makes use of four interrelated curves (hereafter referred to as the "washability curves"). These curves were explained by Bird in a paper[6] published in 1928, but his explanation has been regarded by some as not sufficiently complete to be understood by anyone not thoroughly familiar with the older systems of interpretation.

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[1] The Bureau of Mines will welcome reprinting of this paper, provided the following footnote acknowledgment is used: "Reprinted from Bureau of Mines Information Circular 7045."

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[5] Gandrud, B. W., metallurgist, Southern Experiment Station, Bureau of Mines, Tuscaloosa, Ala.

[6] Bird, B. M., Interpretation of Float-and-Sink Data: Published in Proc., Second International Conference on Bituminous Coal, November 1928, vol. 2, pp. 82-111.

This paper has been prepared in response to requests for a detailed, elementary interpretation of the float-and-sink data. As an essential part of this explanation, the construction of the curves is discussed. The author hopes that this paper will lead to better understanding of reports on washability of coal, particularly those issued by the Bureau of Mines.

CALCULATING THE FLOAT-AND-SINK DATA

The methods and apparatus employed in making the float-and-sink tests have been described in former Bureau of Mines publications[7], [8] and will not be discussed here. The products resulting from the float-and-sink separations, after they have been washed and dried, are weighed and analyzed for moisture and ash. The weights are calculated to percentages and the ash analyses to percentages on the moisture-free basis. These data are tabulated as shown in the first five columns of table 1. The values in column 6, headed "Cumulative weight, percent", are in each instance the sum of all the preceding weight percentages. For example, the first value recorded in the "cumulative weight, percent" column is the same as the first value in the "weight, percent" column; the second value is the sum of the first two weight percentages; the third is the sum of the first three; and so on.

The values listed in column 7 of table 1 have been computed and represent, in each instance, the ash analysis of the total float coal on the corresponding specific gravity shown in column 2. For instance, the total coal floating at 1.27 specific gravity analyzed 2.8 percent ash; at 1.30 specific gravity, the cumulative ash analysis would be 3.3 percent; at 1.38, the cumulative ash would be 4.5 percent; and so on. The last value, 12.9 percent, would be the analysis of the total coal sample, including the sink in the liquid of 1.90 specific gravity. The calculation of the cumulative ash percentage is based on the equation $\frac{\text{"weight, percent" x "ash, percent"}}{100}$ = units of ash, where "units" means parts in the number of parts expressed by the corresponding weight percentage.

Referring again to the data of table 1, the cumulative ash for the float-on-1.27 fraction is the same as the corresponding percentage listed under "ash, percent." The next cumulative ash value may be calculated in the following manner: In the float-on-1.27 fraction there is $\frac{34.5 \text{ x } 2.8}{100}$ or 0.9660 unit of ash; in the 1.27-1.30 fraction there are $\frac{28.4 \text{ x } 3.9}{100}$ or

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[7] McMillan, Earl R., and Bird, B. M., A Float-and-Sink Method and Apparatus for Testing Coarse Size Coal: Rept. of Investigations 2570, Bureau of Mines, 1924, 12 pp.

[8] Bird, B. M., and Messmore, H. E., The Float-and-Sink Test for Fine Coal: Rept. of Investigations 2586, Bureau of Mines, 1924, 8 pp.

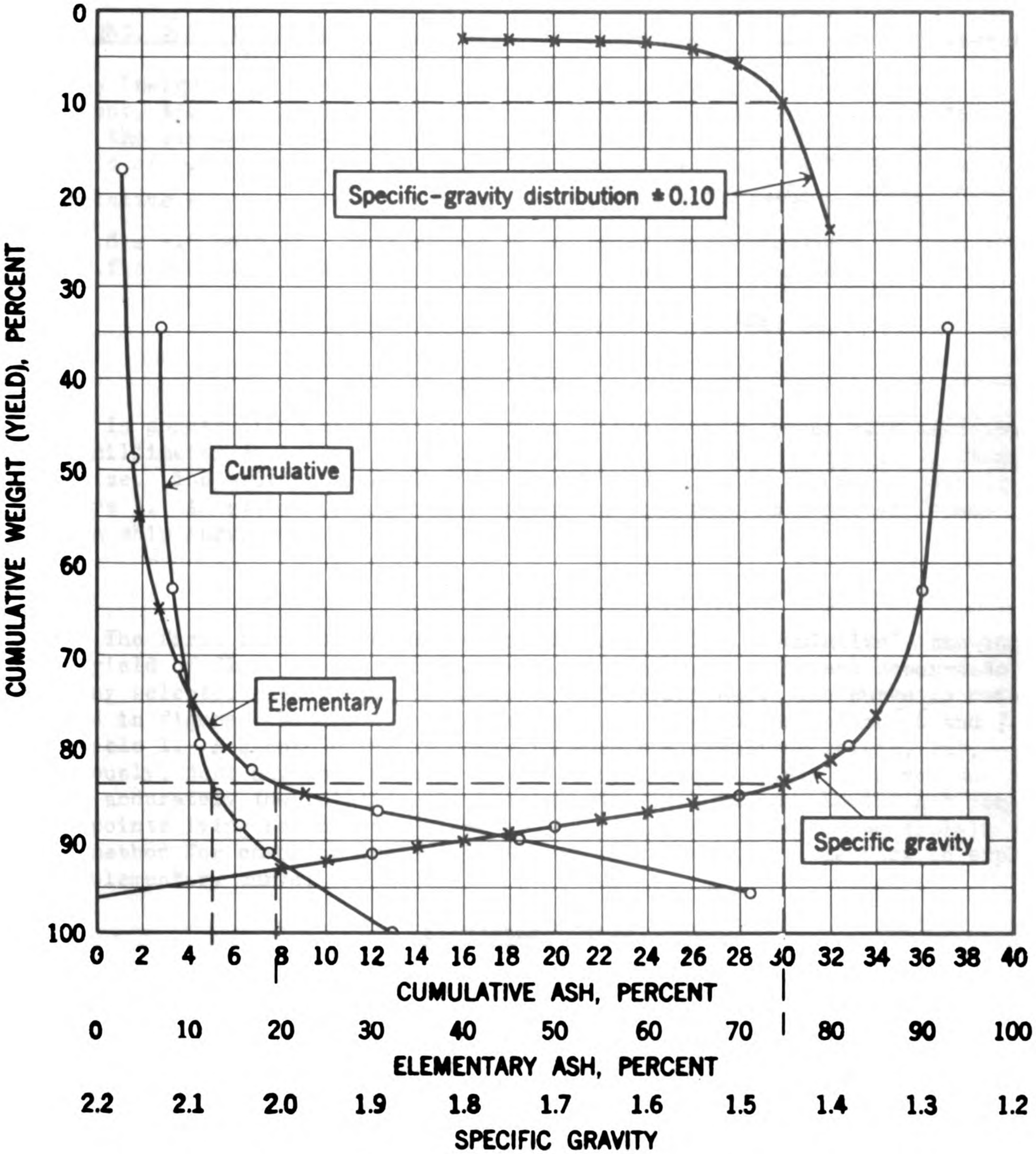

Figure 1.—Coal-washability curves.

1.1076 units of ash. The sum of these, or 2.0736, is the units of ash in the total material lighter than 1.30 specific gravity, which, as shown by column 6, comprises 62.9 percent by weight of the sample. Since $\frac{\text{"weight, percent" x "ash, percent"}}{100} = \text{"units of ash"}$, then "units of ash" x 100 ÷ "weight, percent" = "ash, percent"; and 2.0736 x 100 ÷ 62.9 = 3.3 percent, the average ash content of the float-on-1.27 fraction combined with the sink-on-1.27-float-on-1.30 fraction, or the total float on the liquid of 1.30 specific gravity. The calculations for the third recorded cumulative ash percentage are: $\left[\frac{(34.5 \text{ x } 2.8)}{100} + \frac{(28.4 \text{ x } 3.9)}{100} + \frac{(16.9 \text{ x } 8.8)}{100}\right] 100 \div 79.8 = 4.5$ percent. This system of calculation is continued for all of the specific-gravity fractions down to and including the sink in 1.90.

CONSTRUCTION OF THE WASHABILITY CURVES

Equipment

In constructing washability curves, cross-section paper with centimeter and millimeter divisions is used. This paper must be at least 21 by 25 cm in size. The ordinate and abscissa scales should be in the form shown in figure 1. An almost indispensable piece of equipment is a No. 48 Copenhagen ship curve.

Cumulative Curve

The first curve to be plotted is the one called "cumulative", showing the yield of float coal resulting from a 100-percent efficient separation at any selected cumulative, or average, ash percentage. The curve is outlined in figure 1 by plotting the percentages found under columns 6 and 7 in table 1. A smooth curve is drawn through the resulting points, but, obviously, just any smooth curve drawn through these points does not depict accurately the relationship between the yield and cumulative ash for all points lying between the known or plotted points. Before we explain the method for checking the cumulative curve, it will be necessary to explain the elementary curve.

Elementary Curve

Mathematically, the elementary curve is the derivative of the cumulative curve and shows the rate of change of the ash content at different yields. In other words, the elementary curve is intended to show the ash percentage in the highest ash particle included in a float coal product of any given cumulative ash percentage. It is impracticable to make use of the calculus to calculate this curve because of the inordinate amount of time necessary to determine and differentiate the equation for the cumulative curve. However, the elementary curve can be established with reasonable accuracy by the following method, which employs no mathematics higher than arithmetic. It should be understood that this is a convenient approximation rather than a mathematically exact method.

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Columns 4 and 5 in table 1 show that 34.5 percent of the total coal is of lower specific gravity than 1.27 and that the average ash content of this product is 2.8 percent. Obviously, every particle of coal included in this product does not contain exactly 2.8 percent ash. The analysis does not show what the ash range is, but merely that these particles of coal collectively contain 2.8 percent ash. The reasoning involved in determining the ash percentage of a particle of any definite specific gravity is illustrated by figure 2. Assume that the vertical column in this figure represents the coal comprising the lower specific-gravity fractions arranged in order of increasing specific gravity, with the particle of lowest specific gravity at the bottom. The particle containing the smallest amount of ash is represented at 0.0 percent cumulative weight, and the particle of highest ash in the float-on-1.27 fraction would, according to the data of table 1, reach a point in the column corresponding to 34.5 percent cumulative weight. Since the average ash content of all the particles included in the range between 0.0 and 34.5 percent cumulative weight is 2.8 percent, and since the particles are arranged in order of increasing ash content, assume that all of the particles below the midpoint, or 17.25 percent cumulative weight, contain less than 2.8 percent ash and all of those higher than 17.25 percent cumulative weight contain more than 2.8 percent ash. From this it follows that the particle represented at 17.25 should contain 2.8 percent ash. Therefore, $\frac{34.5}{2}$, or 17.25, plotted at 2.8 percent elementary ash is the first known point on the elementary curve. Table 1 shows that the particle having a specific gravity of exactly 1.30 should be represented in the column at a point 28.4 percent higher on the weight scale than the particle of exactly 1.27 specific gravity, or at a cumulative weight of 62.9 percent. The particles in this 28.4-weight-percent range average 3.9 percent ash. Here, again, we assume that the particle located midway of the range contains exactly the same amount of ash as the average for this range, or 3.9 percent. The cumulative weight percentage corresponding to 3.9 percent elementary ash is $\frac{28.4}{2} + 34.5$, or 48.7. Thus, the second known point on the elementary curve is located by plotting 48.7 at 3.9 percent elementary ash. A rule for calculating points on the elementary curve directly from the float-and-sink data may be expressed as follows:

> One-half of the "weight, percent" of the specific-gravity interval involved, plus the "cumulative weight, percent" of all material of lower specific gravity, is plotted against the ash content (not cumulative ash) of the specific gravity interval involved.

Application of this formula to the values recorded in table 1 results in the following calculations:

$\frac{34.5}{2}$ or 17.25 percent cumulative weight plotted at 2.8 percent elementary ash.

$\frac{28.4}{2}$ or 14.2 plus 34.5 percent = 48.7 percent cumulative weight plotted at 3.9 percent elementary ash.

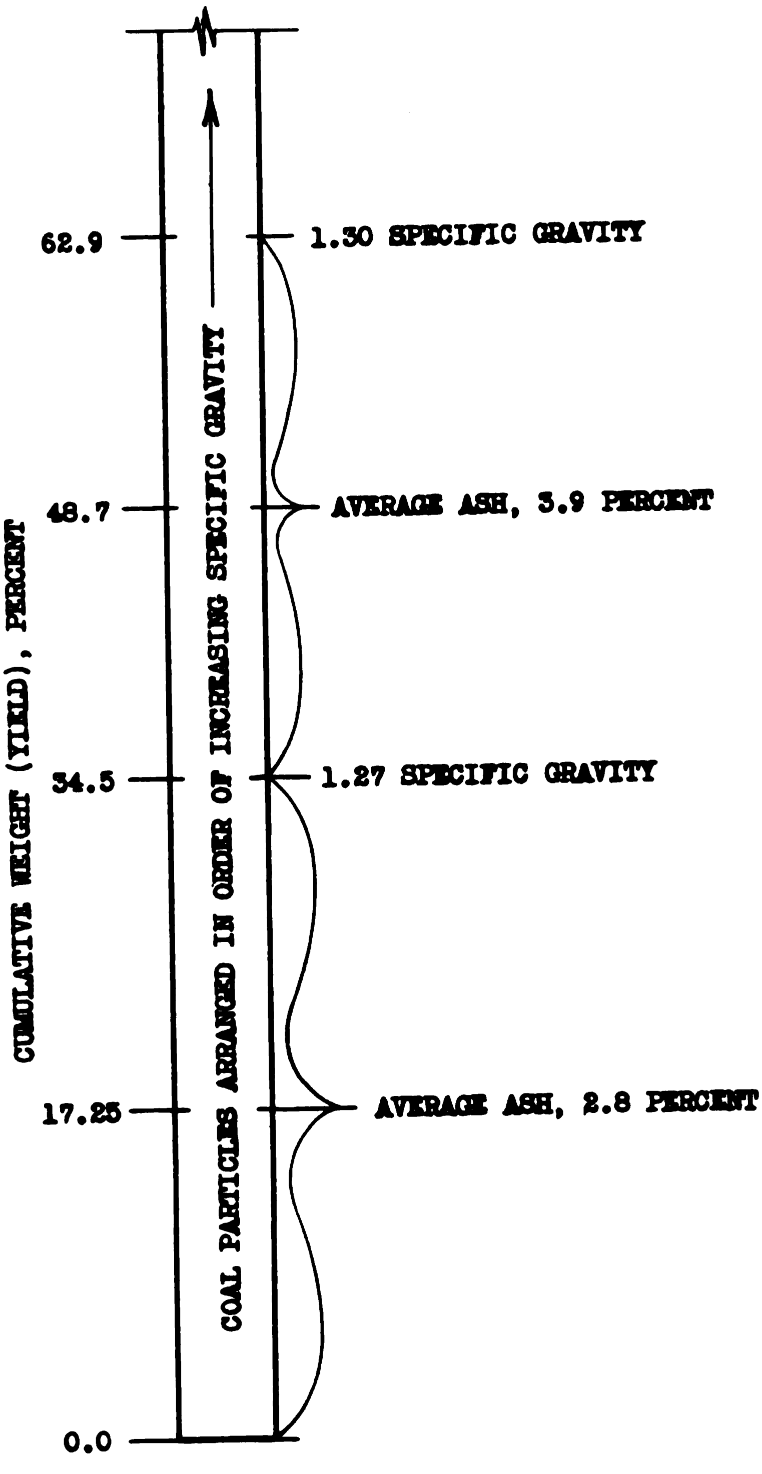

Figure 2.—Elementary-ash chart.

$\frac{16.9}{2}$ or 8.45 plus 62.9 = 71.35 percent cumulative weight plotted at 8.8 percent elementary ash.

$\frac{5.4}{2}$ or 2.7 plus 79.8 = 82.5 percent cumulative weight plotted at 16.9 percent elementary ash.

$\frac{3.3}{2}$ or 1.65 plus 85.2 = 86.85 percent cumulative weight plotted at 30.6 percent elementary ash.

$\frac{3.0}{2}$ or 1.5 plus 88.5 = 90.0 percent cumulative weight plotted at 46.2 percent elementary ash.

$\frac{8.5}{2}$ or 4.25 plus 91.5 = 95.75 percent cumulative weight plotted at 71.3 elementary ash.

Thus, seven points are established, which serve to outline the elementary curve.

Checking Cumulative Curve by Means of Elementary Curve

Preferably, the float-and-sink separations are made on liquids of specific gravities that will give points that are closely spaced. This is especially desirable in the range where the curve would change direction rapidly. When this is done and the curve is smooth through each of the plotted points there is little likelihood of drawing the cumulative curve incorrectly. Nevertheless, the position of the cumulative curve between the plotted points should always be checked by means of the elementary curve.

In making this check, the difference in units of ash at two yield percentages, calculated from the cumulative curve, is divided by the difference in these yield percentages. The resulting value is the elementary ash at a yield midway between the selected yields. In a range where the cumulative curve changes direction slowly or not at all, the yields usually selected are at 10-percent intervals; but in those ranges where the cumulative curve changes direction rapidly, a smaller increment than 10 percent must be used. For example, in figure 1, if it is desired to determine the elementary-ash percentage at 55 percent yield where the change is slow, the calculation is:

$\frac{60\% \text{ yield} \times 3.19\% \text{ cumulative ash}}{100} = 1.914$ units of ash,

$\frac{50\% \text{ yield} \times 2.90\% \text{ cumulative ash}}{100} = 1.450$ units of ash,

10 percent yield difference at 0.464 unit of ash difference,

$\frac{0.464 \times 100}{10} = 4.64$ percent elementary ash at 55 percent yield.

But if the elementary-ash percentage is desired at 85 percent yield where the change is rapid, the increment should be smaller, as illustrated by the following calculation:

$$\frac{87\%\text{ yield x }5.72\%\text{ cumulative ash}}{100} = 4.976\text{ units of ash,}$$

$$\frac{83\%\text{ yield x }4.90\%\text{ cumulative ash}}{100} = 4.067\text{ units of ash,}$$

4 percent yield difference at 0.909 units of ash difference,

$$\frac{0.909\text{ x }100}{100} = 22.7\text{ percent elementary ash at 85 percent yield.}$$

In this manner enough intermediate points are plotted to establish accurately the shape of the elementary curve. The calculations for the remaining intermediate elementary points shown in figure 1 are:

To find the elementary-ash percentage at 65 percent yield:

$$\frac{70\%\text{ x }3.70\%}{100} = 2.590\text{ units of ash,}$$

$$\frac{60\%\text{ x }3.19\%}{100} = 1.914\text{ units of ash,}$$

10 percent difference in yield corresponds to 0.676 unit of ash,

$$\frac{0.676\text{ x }100}{10} = 6.76\text{ percent elementary ash at 65 percent yield.}$$

To find the elementary ash percent at 75-percent yield:

$$\frac{80\%\text{ x }4.52\%}{100} = 3.616\text{ units of ash,}$$

$$\frac{70\%\text{ x }3.70\%}{100} = 2.590\text{ units of ash,}$$

10 percent difference in yield corresponds to 1.026 units of ash,

$$\frac{1.026\text{ x }100}{10} = 10.26\text{ elementary ash at 75 percent yield.}$$

To find the elementary ash percent at 80 percent yield:

$$\frac{85\%\text{ x }5.25\%}{100} = 4.463\text{ units of ash,}$$

$$\frac{75\%\text{ x }4.08\%}{100} = 3.060\text{ units of ash,}$$

10 percent difference in yield corresponds to 1.403 units of ash,

$$\frac{1.403\text{ x }100}{10} = 14.03\text{ percent elementary ash at 80 percent yield.}$$

Figure 1 shows that a curve passing through all of the plotted elementary points is smooth and regular, from which we conclude that the cumulative curve is correctly drawn. Had the cumulative curve been improperly drawn, as it is in figure 3, the elementary curve, instead of being smooth, would have been irregular, with sharp changes in rate of curvature. Since

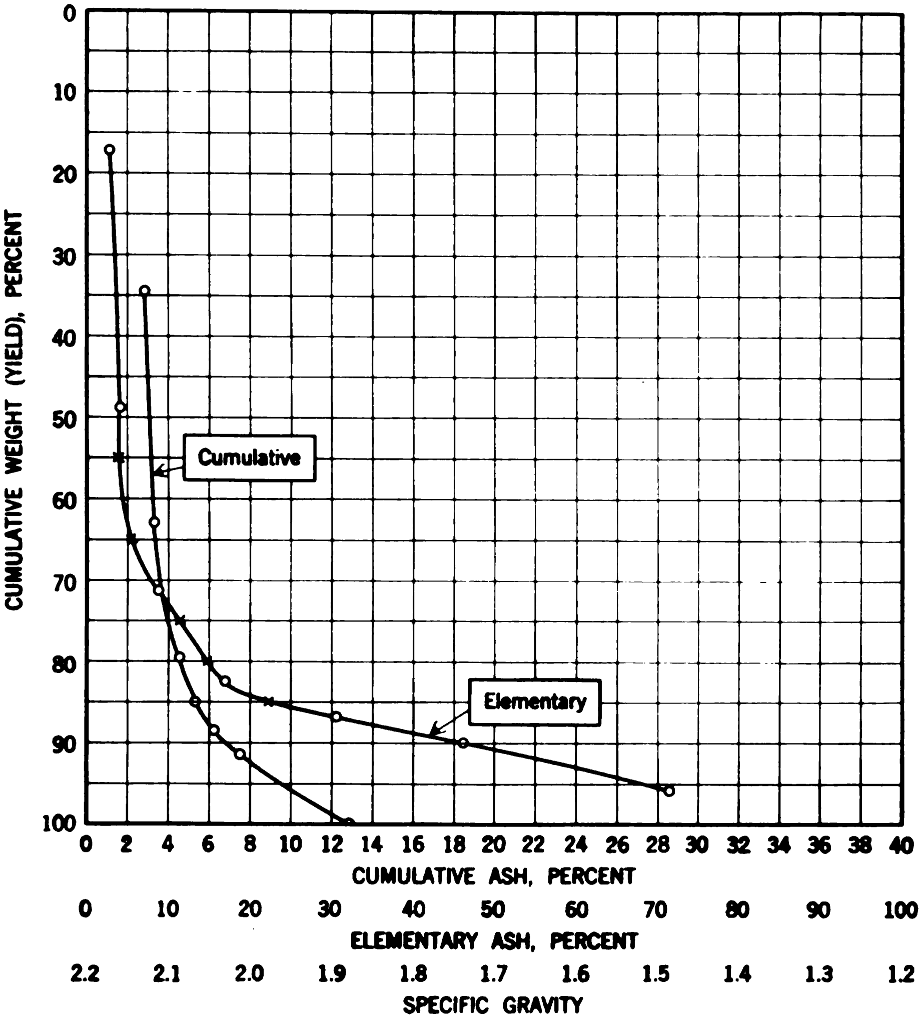

Figure 3.—Example of cumulative curve incorrectly drawn.

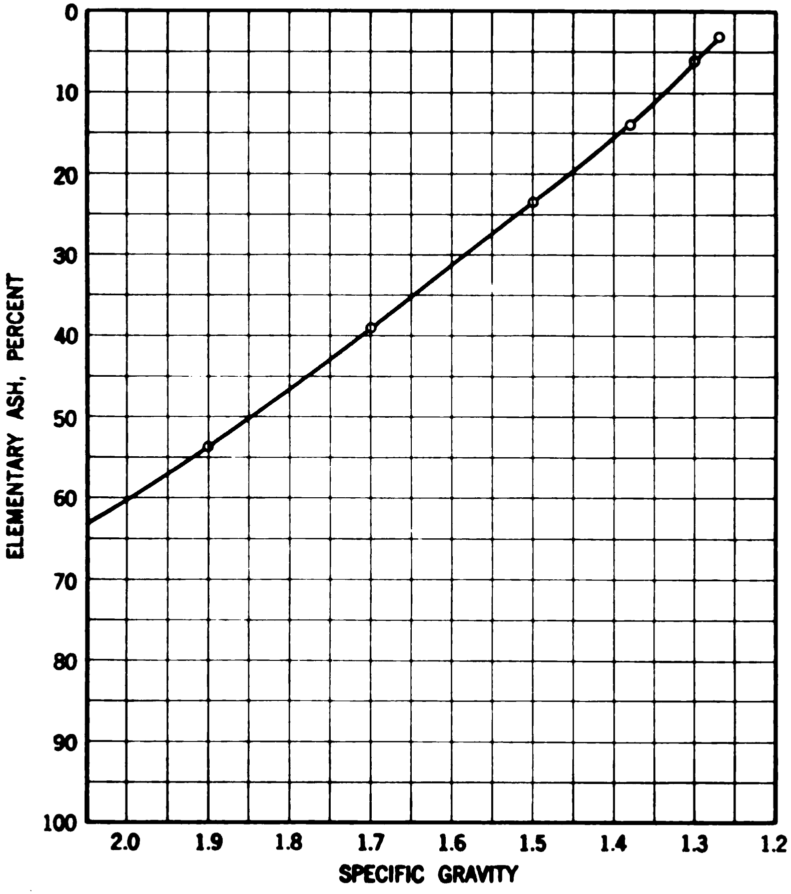

Figure 4.—Elementary ash-specific gravity curve.

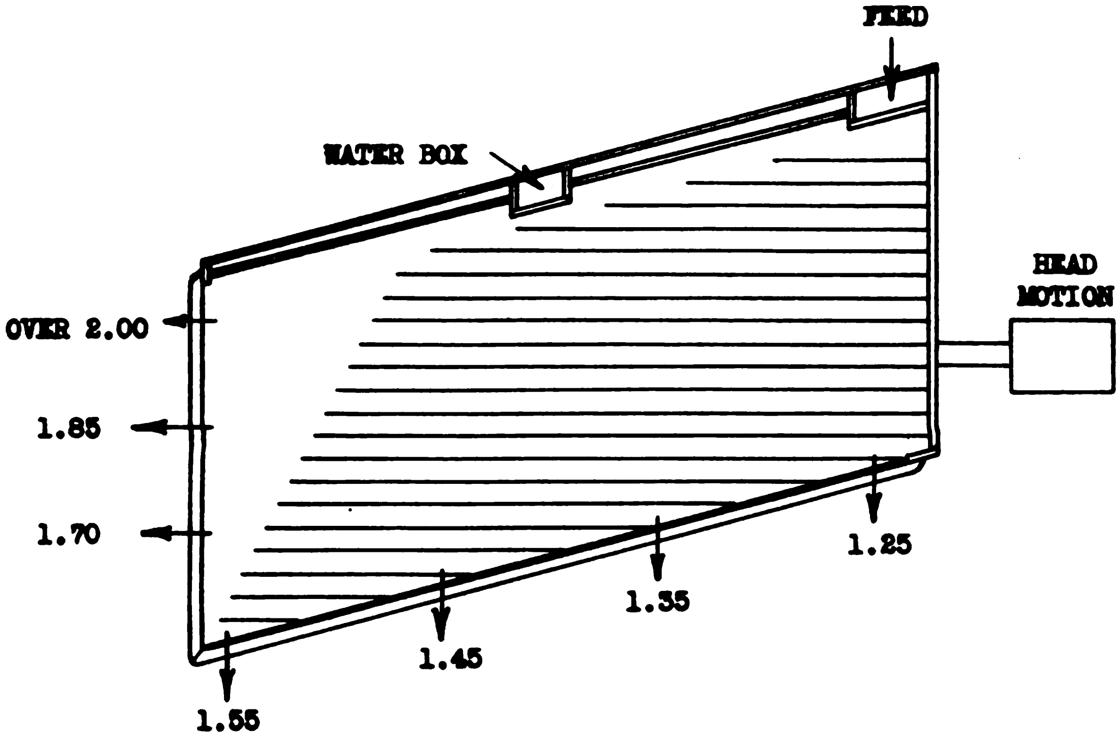

Figure 5.—Illustration of separations according to specific gravities on a wet concentrating table.

the elementary curve shows the rate of change of the cumulative curve, it should be a smooth curve containing no sharp bends or sharp reversals. When the cumulative curve is drawn so that elementary points calculated therefrom form a curve that complies with these requirements, we may consider that the cumulative curve is correctly drawn. Figure 3 is included merely to show how a slight change in the shape of the cumulative curve will affect the shape of the elementary curve.

Specific-Gravity Curve

The specific gravity curve in figure 1 shows the yield of float coal for a perfect separation at any specific gravity within the range of gravities of the float-and-sink tests.

This curve is constructed by plotting the specific gravities listed in column 2 of table 1 against the corresponding cumulative weight percentage, column 6. Intermediate points are found in the following manner: At the yield corresponding to each specific gravity at which a float-and-sink separation was made, the elementary ash percent is read on the elementary curve, and this value is plotted on a separate sheet of coordinate paper (see fig. 4) against the specific gravity to which it corresponds. It is assumed that the ash content is a first-degree function of the specific gravity. If this is true and if the elementary curve is correctly drawn, these elementary ash-specific gravity points will be in a straight line. The elementary ash-specific gravity curve is rarely a straight line, but, on the other hand, it is seldom that a coal is found that will deviate much from this assumption. The points thus plotted are connected by a smooth curve, which approaches as nearly as possible a straight line and passes through each of the plotted points. From this curve, the elementary ash values are read at intervals of about 0.05 specific gravity unit. Each elementary ash value thus read from the auxiliary curve is located on the elementary curve, and the yield to which this corresponds is plotted at the selected specific gravity. For example, figure 4 shows at 1.40 specific gravity an elementary ash percentage of 15.6. This value, read on the elementary curve of figure 1, corresponds to 81.4 percent yield. At 1.40 specific gravity the yield 81.4 percent is plotted. In this manner, a series of intermediate points is found, which, together with the points plotted directly from the float-and-sink data, are connected to form a smooth curve.

The Plus-and-Minus 0.10 Specific Gravity-Distribution Curve

The ±0.10 specific gravity-distribution curve in figure 1 shows the percentage by weight of the coal that lies within plus 0.10 and minus 0.10 specific-gravity unit at any given specific gravity. For instance, the ±0.10 value at 1.40 specific gravity is the percentage of the total coal that lies within the 1.30 to 1.50 specific gravity range. At 1.45 specific gravity, the ±0.10 value is the percentage between 1.35 and 1.55 specific gravity, and so on. The connection between ±0.10 values and difficulty of effecting a separation between coal and refuse is illustrated by figure 5, which represents a wet concentrating table. Theoretically, if the feed to a table consists of particles of uniform size and shape, the particles of lowest specific gravity will be discharged from the side of the table near the head-motion end, the particles of highest specific gravity will be discharged at the opposite end of the table at a point beyond the water box, and the particles of intermediate specific gravities will be discharged along the side and end at points progressively spaced according to specific gravity. For sake of illustration, specific-gravity values have been assigned at several points of discharge on the table represented in figure 5. Actually, much overlapping occurs, and the separation at any point along the table is not sharp. For example, if a separation is made at the point indicated as representing 1.45 specific gravity, there will be included in the washed coal some particles of higher specific gravities and in the refuse some particles of lower specific gravities than 1.45. Most of the overlapping occurs within the range of plus and minus 0.10 specific gravity unit from the specific gravity at which the separation is made. If only a few percent by weight of the table feed are discharged within the 1.35 to 1.55 range, a highly efficient separation can be made at 1.45 specific gravity and there will be little difficulty in maintaining a constant ash percentage in the washed coal. Such a separation would be classed as a simple problem. But if half of the feed to the table is discharged within the 1.35 to 1.55 specific gravity range, it will not be feasible to make a separation at 1.45 specific gravity, such a separation being practically impossible. Experiments both with jigs and wet tables have demonstrated that if not more than 10 percent of the coal is within the ±0.10 specific gravity range at the point of separation, either type of machine is capable of effecting an efficient separation on a size range to which the machine is adapted. Thus, it is apparent that by use of the ±0.10 specific gravity-distribution curve in conjunction with the other washability curves, it is possible to predict with reasonable accuracy the grade and yield from any coal for which washability curves are available.

Further comments about the plus-and-minus 0.10 specific gravity-distribution curve are required. It is constructed in the following manner: The yield at 1.30 specific gravity is subtracted from the yield at 1.50 specific gravity as read from the specific gravity curve in figure 1. To compensate for varying amounts of high-gravity materials, especially slate and other rock, the numerical difference in the yields is divided by the yield at 2.00 specific gravity. The resulting adjusted percentage is plotted at 1.40 specific gravity. The reason for dividing the difference in the two yields by the yield at 2.00 specific gravity is that the material of higher specific gravity than 2.00, because of its rapid settling rate, would not interfere with the separation between washed coal and refuse at normal specific gravities. Failure to make this correction could result in the absurd condition where the addition of roof rock to the washery feed would apparently decrease the difficulty of the separation because it would decrease the percentage of material within the ±0.10 range. The next point is determined by subtracting the yield at 1.35 specific gravity from the yield at 1.55 specific gravity. This difference, divided by the yield at 2.00 specific gravity, is plotted at 1.45 specific gravity. In this manner points are plotted at specific-gravity intervals of 0.05 throughout the range from 1.40 to 1.80 specific gravity. The calculations for the points outlining the specific gravity-distribution curve of figure 1 are:

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Yield at 1.50 sp.g. = 85.2%
Yield at 1.30 sp.g. = 62.9%
85.2 - 62.9 = 22.3
22.3 x 100 ÷ 93.1 (yield at 2.00 sp.g.) = 24.0% or ±0.10 value at 1.40 sp.g.

Yield at 1.55 sp.g. = 86.1%
Yield at 1.35 sp.g. = 76.5%
86.1 - 76.5 = 9.6
9.6 x 100 ÷ 93.1 (yield at 2.00 sp.g.) = 10.3% at 1.45 sp.g.

Yield at 1.60 sp.g. = 86.9%
Yield at 1.40 sp.g. = 81.4%
86.9 - 81.4 = 5.5
5.5 x 100 ÷ 93.1 = 5.9% at 1.50 sp.g.

Yield at 1.65 sp.g. = 87.8%
Yield at 1.45 sp.g. = 83.9%
87.8 - 83.9 = 3.9
3.9 x 100 ÷ 93.1 = 4.2% at 1.55 sp.g.

Yield at 1.70 sp.g. = 88.5%
Yield at 1.50 sp.g. = 85.2%
88.5 - 85.2 = 3.3
3.3 x 100 ÷ 93.1 = 3.5% at 1.60 sp.g.

Yield at 1.75 sp.g. = 89.3%
Yield at 1.55 sp.g. = 86.1%
89.3 - 86.1 = 3.2
3.2 x 100 ÷ 93.1 = 3.4% at 1.65 sp.g.

Yield at 1.80 sp.g. = 90.1%
Yield at 1.60 sp.g. = 86.9%
90.1 - 86.9 = 3.2
3.2 x 100 ÷ 93.1 = 3.4% at 1.70 sp.g.

Yield at 1.85 sp.g. = 90.9%
Yield at 1.65 sp.g. = 87.8%
90.9 - 87.8 = 3.1
3.1 x 100 ÷ 93.1 = 3.3% at 1.75 sp.g.

Yield at 1.90 sp.g. = 91.5%
Yield at 1.70 sp.g. = 88.5%
91.5 - 88.5 = 3.0
3.0 x 100 ÷ 93.1 = 3.2% at 1.80 sp.g.

METHOD OF READING THE WASHABILITY CURVES

Due to the fact that all of the curves have a common ordinate, values from one of the curves may be expressed in terms of any of the others. This is illustrated by the broken lines in figure 1. Assume that the coal to which the curves of figure 1 apply is of a size range suitable for table concentration. A reading of 10 percent on the ±0.10 specific gravity-distribution curve represents the normal maximum difficulty at which a wet table is capable of effecting an efficient separation. At 10 percent cumulative weight, figure 1 shows a horizontal broken line that intersects the ±0.10 curve at 1.452 specific gravity. The vertical broken line at this specific gravity intersects the specific gravity curve at 83.9 percent cumulative weight, and the horizontal broken line at 83.9 percent cumulative weight is shown to intersect the elementary curve at 19.5 percent ash and the cumulative curve at 5.0 percent ash. In other words, the curves predict that a wet table, if expertly operated and if other conditions are favorable, should be capable of washing this coal efficiently to 5.0 percent ash with a theoretical yield of 83.9 percent. Included in the washed coal would be particles containing as high as 19.5 percent ash. The efficiency of the separation is the ratio of the actual yield to the theoretical or float-and-sink yield, and should be about 95 percent. It is not unusual for a table to operate at 97 to 98 percent efficiency, but 95 percent represents the usual average when the object is to produce as clean a washed coal as possible. Thus, the actual yield of 5.0 percent ash washed coal that could be expected is 95 percent of 83.9, or 79.7 percent of the total raw coal feed.

TABLE 1. - Arrangement of float-and-sink data

Description (1)	Specific gravity (2)	Weight, kilograms (3)	Weight, percent (4)	Ash[1/], percent (5)	Cumulative weight, percent (6)	Cumulative ash[1/], percent (7)
Coal from the Pratt bed, Warrior field, Alabama.	Float on 1.27	5.10	34.5	2.8	34.5	2.8
	1.27-1.30	4.20	28.4	3.9	62.9	3.3
	1.30-1.38	2.50	16.9	8.8	79.8	4.5
	1.38-1.50	0.79	5.4	16.9	85.2	5.3
	1.50-1.70	0.48	3.3	30.6	88.5	6.2
	1.70-1.90	0.45	3.0	46.2	91.5	7.5
	Sink in 1.90	1.25	8.5	71.3	100.0	12.9
		14.77				

1/ Moisture-free basis.